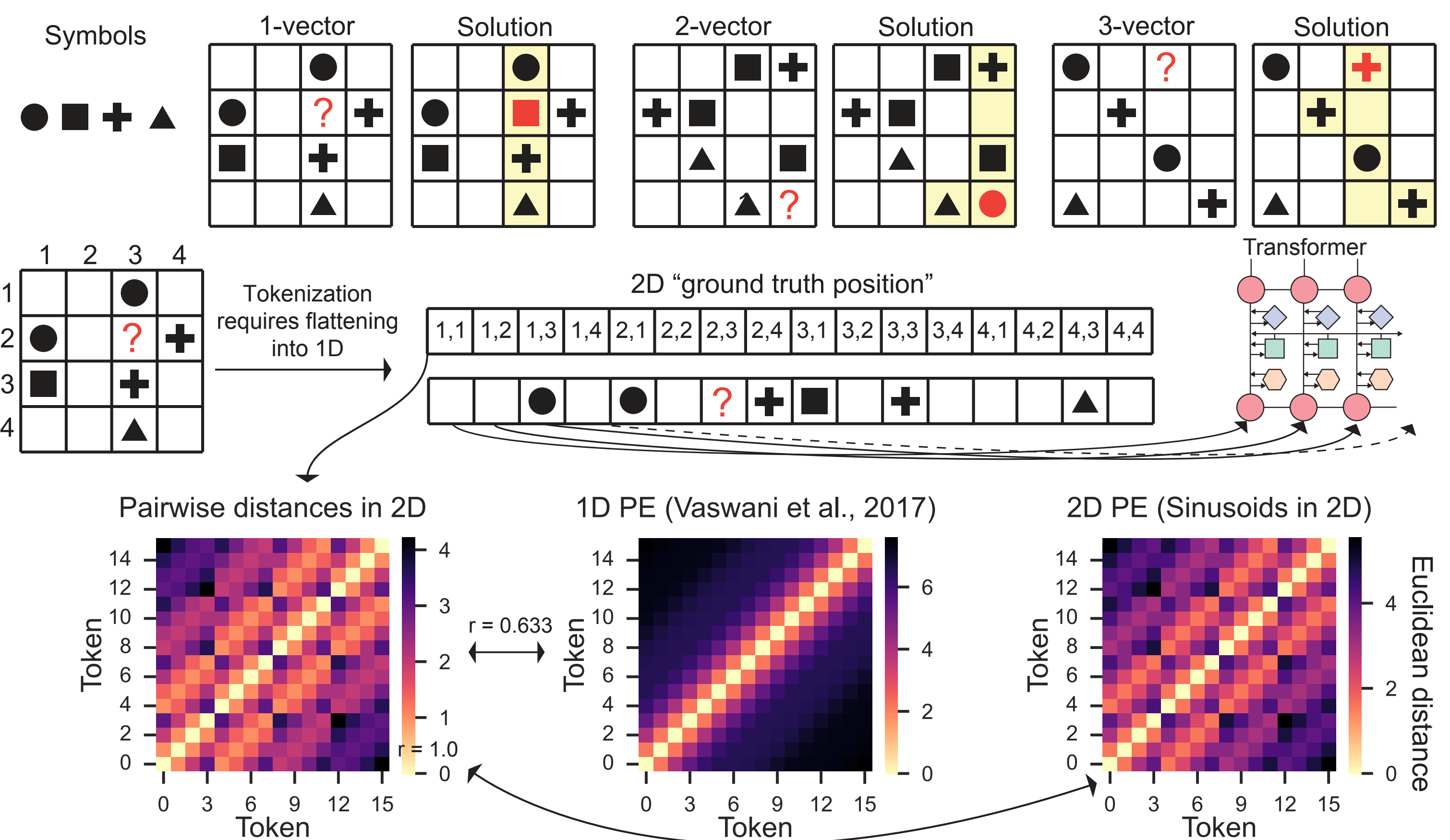


Takuya Ito¹, Luca Cocchi, Tim Klinger, Parikshit Ram, Murray Campbell, Luke J. Hearne^{arXiv}
¹Mathematics of Computation, IBM Research, ²QIMR Berghofer Medical Research Institute, QLD, Australia
 E-mail: taku.ito1@gmail.com

Motivation: The importance of positional encoding choice for transformer generalization

- * Choice of positional encodings (PE) in transformers have been shown to be critical for learning and generalization.
- * Most investigations into PE have been tailored towards 1D string-based tasks, such as arithmetic or context-free grammars using pre-specified PEs (e.g., ROPE, or absolute PEs)
- * Here we investigate the importance on a suite of tasks with sequence data organized in higher dimensions (>greater than 1D sequences)
- * Specifically, we study the conditions by which we can **learn interpretable positional encodings**, and study how they impact generalization
- * Inspired by recent work on rich and lazy representation learning, we explored how initialization of a learnable PE parameter influences interpretability and generalization in transformers

Intuition from a simple 2D task: The Latin Squares Task (simplified Sudoku)

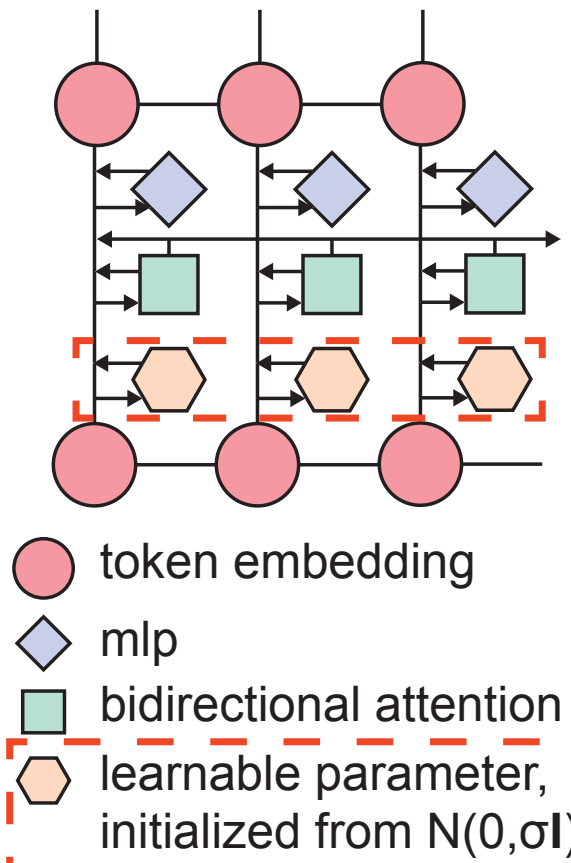


Experiment 1: Learning interpretable positional encodings in the Latin Squares Task

Model manipulation

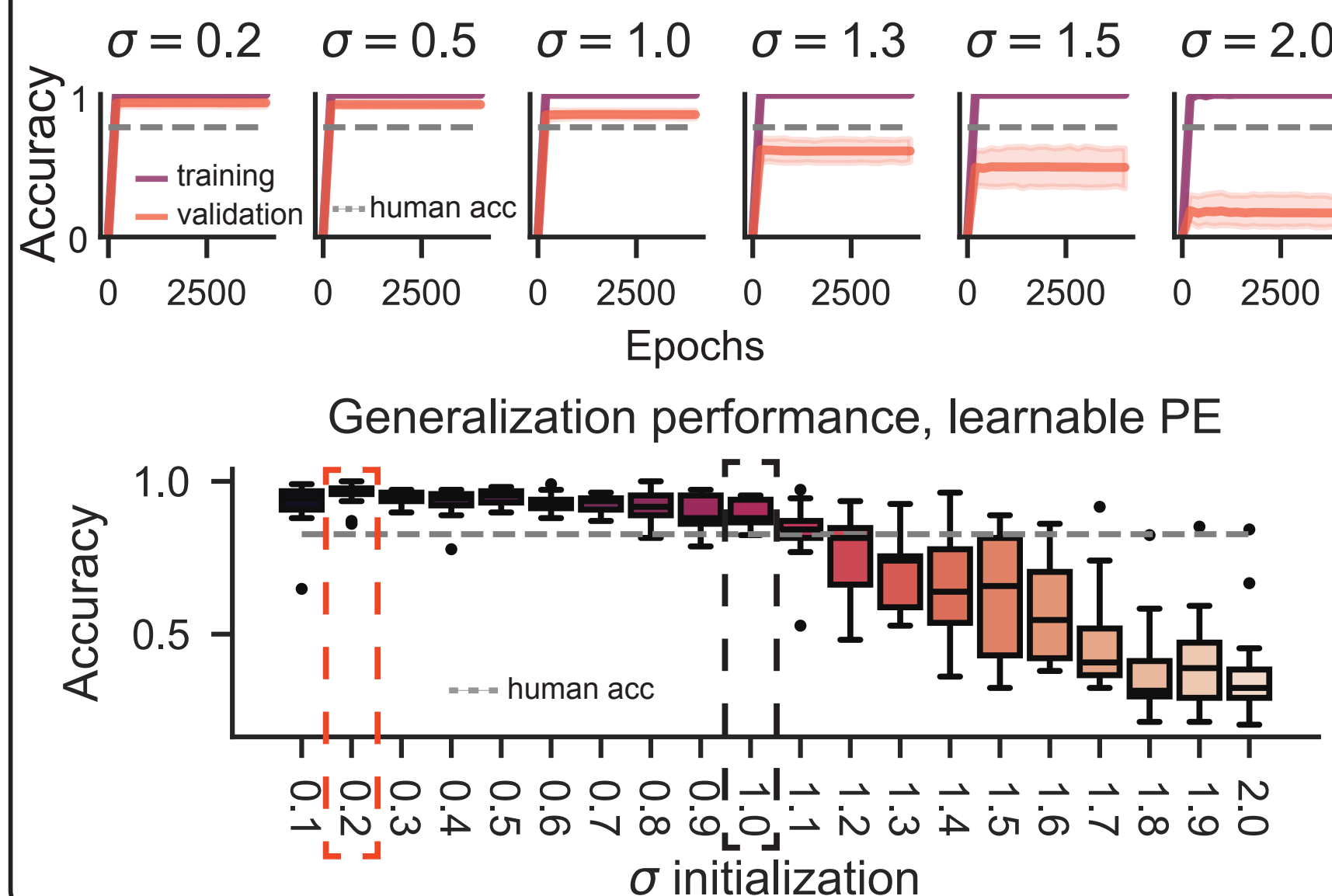
Replace traditional PEs with a learnable parameter initialized from $\mathbf{N}(0, \sigma)$, and manipulate σ

Transformer architecture



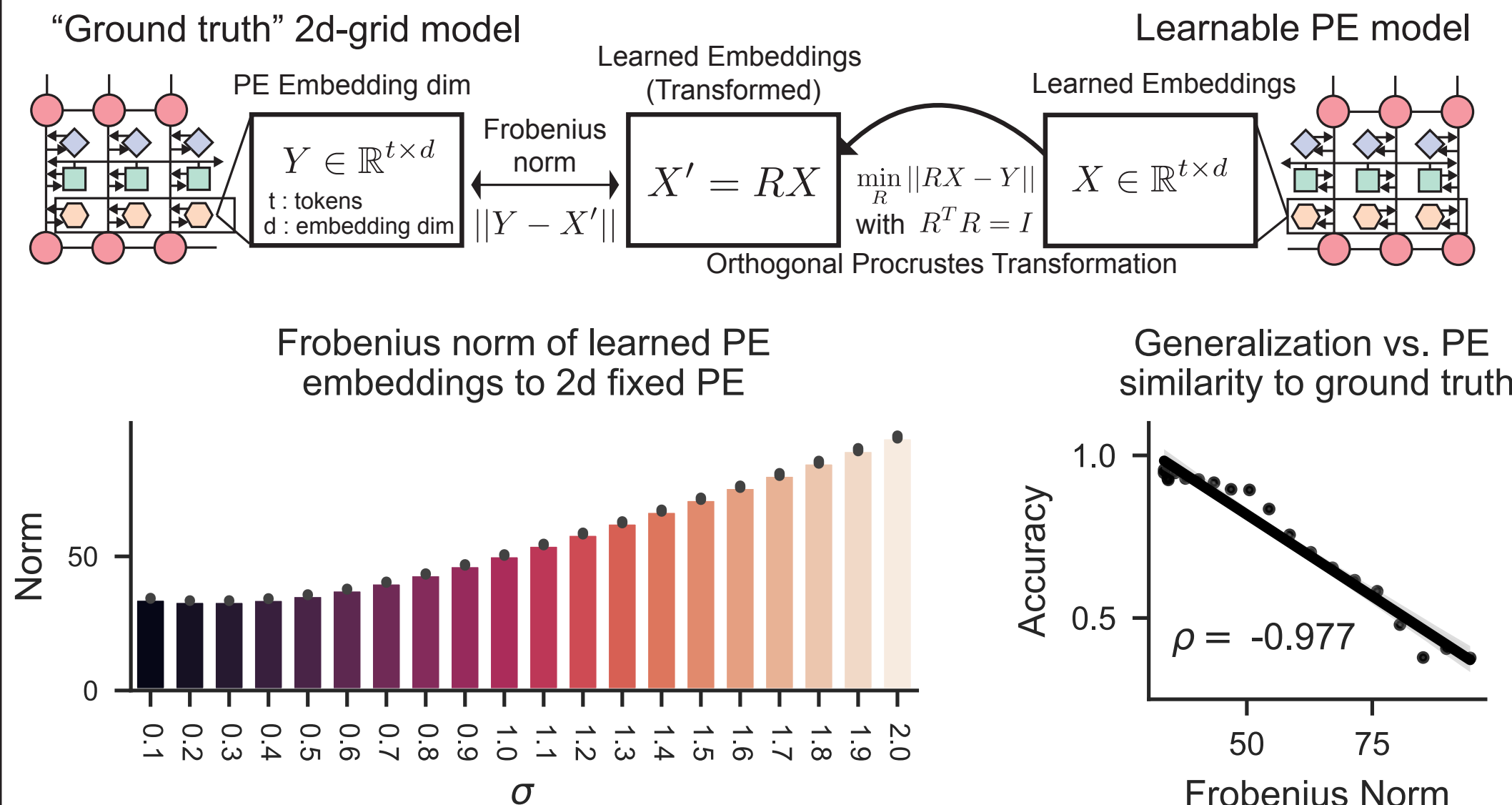
Generalization

Enhanced test set generalization when training a learnable PE embedding initialized from small σ
 Large σ : Lazy learning | Small σ : Rich learning



Interpretability analyses

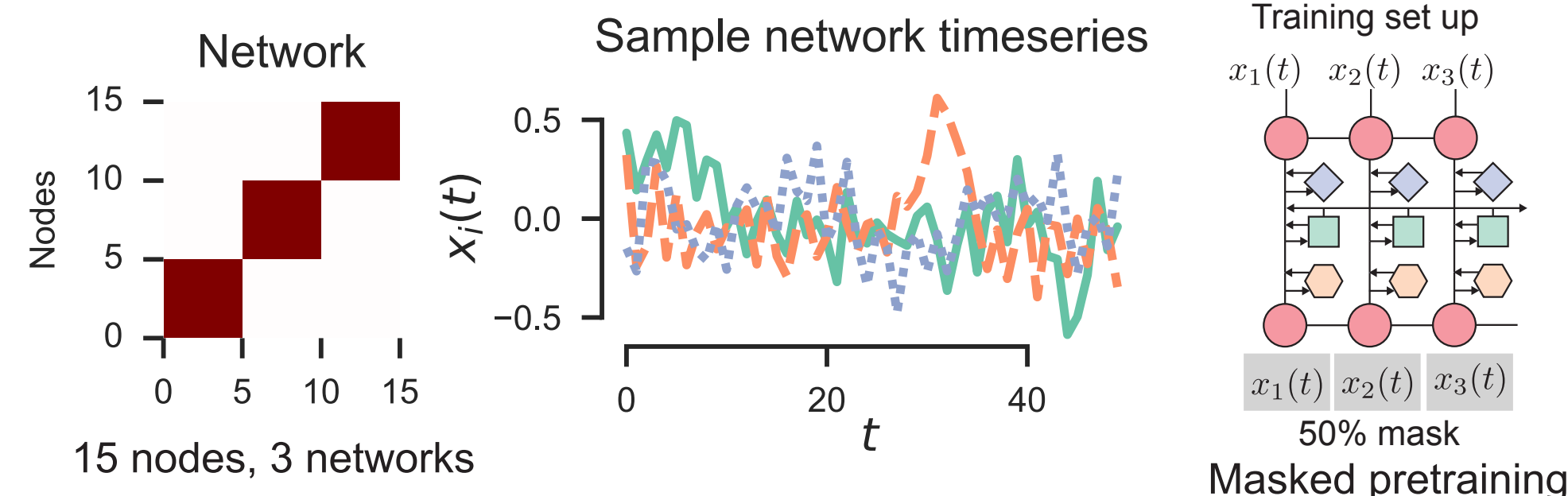
Enhanced interpretability of PE embeddings when using a learnable PE embedding initialized with small σ
 Learned PE embedding mimics a 2D grid structure, consistent with the LST input



Experiment 2: Learnable PEs recover interpretable network clusters in network simulation

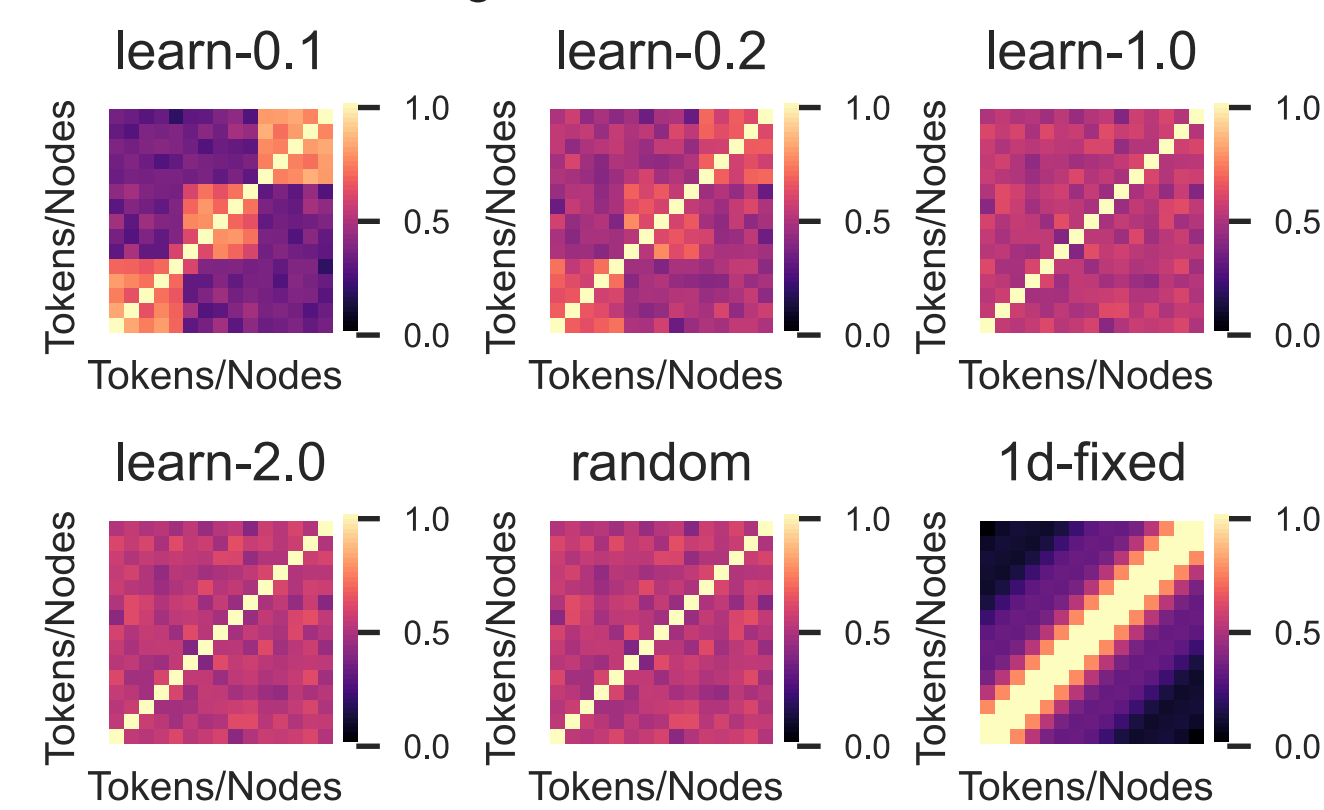
Experimental setup: Simulate nonlinear autoregressive network simulation

$$x_i(t) = \sum_{k=1}^p w_{i,k} \cdot x_i(t-k) + \sum_{j \in C_i, j \neq i} \lambda_{ij} \cdot f(x_j(t-1)) + \sum_{j \notin C_i} \eta_{ij} \cdot f(x_j(t-1)) + \epsilon_i(t)$$



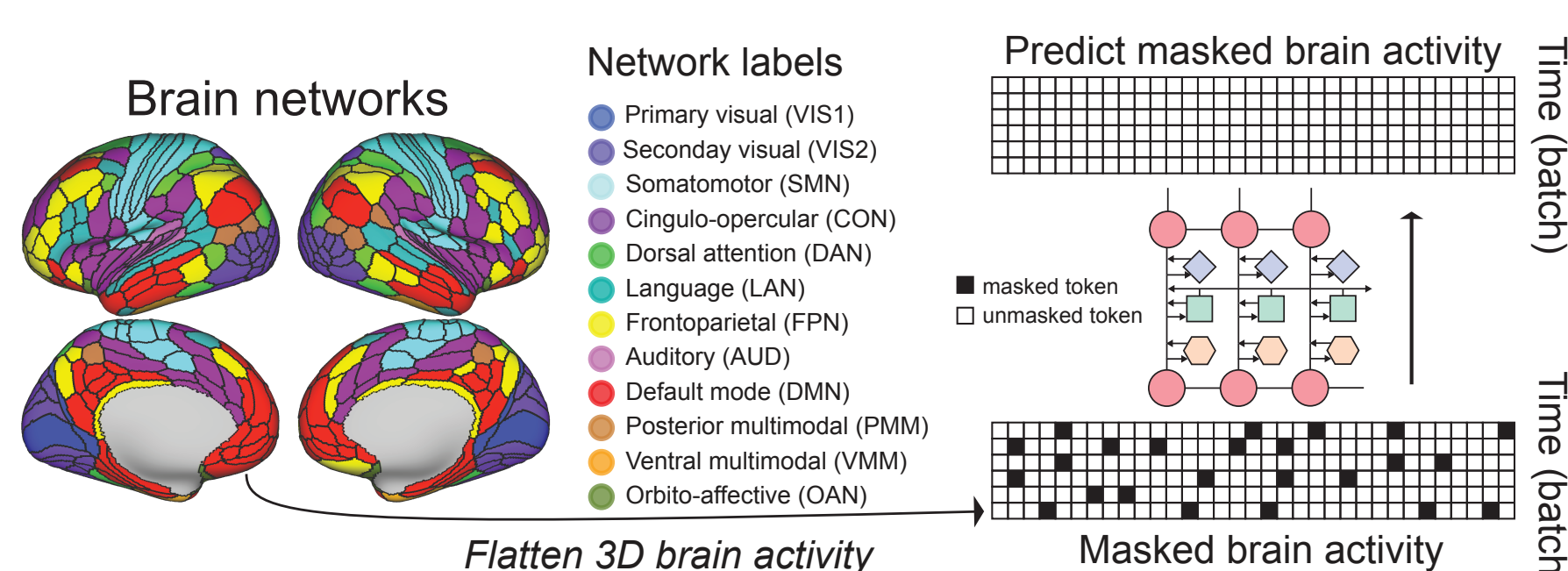
Interpretability analyses: Richly learned PE embeddings recover network clusters

- * We measured the cosine distance of learned PE embeddings across a range of PE models
- * Small σ models (e.g., learn-0.1) accurately recovered the distance between PE embeddings
- * Large σ models (e.g., learn-1.0) did not
- * Models with random and 1d-fixed PEs also did not exhibit network structure

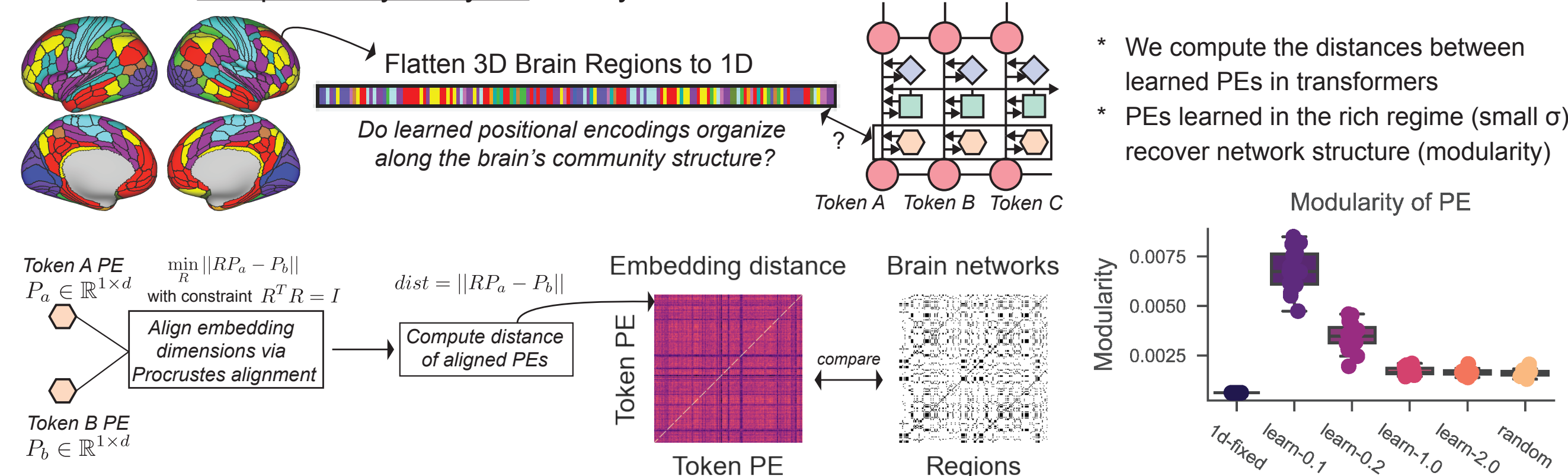


Experiment 3: Learnable PEs recover known network clusters in human brain fMRI data

Experimental setup: Predicting whole-brain activity from masked inputs



Interpretability analyses: Richly learned PEs recover known functional network clusters



Conclusion

- * We extend prior transformer generalization studies from 1D sequences to n-dimensional sequences, which requires positional encoding schemes for higher dimensions.
- * We demonstrate that **rich representation learning of positional encodings** – which is induced by initializing parameters with a small norm – **learns interpretable embeddings that also enhance generalization**